Monomial crystals and promotion operators

Description of the $\mathcal{U}_q(\hat{sl}_4)$-crystal $\mathcal{M}_{2,a}$

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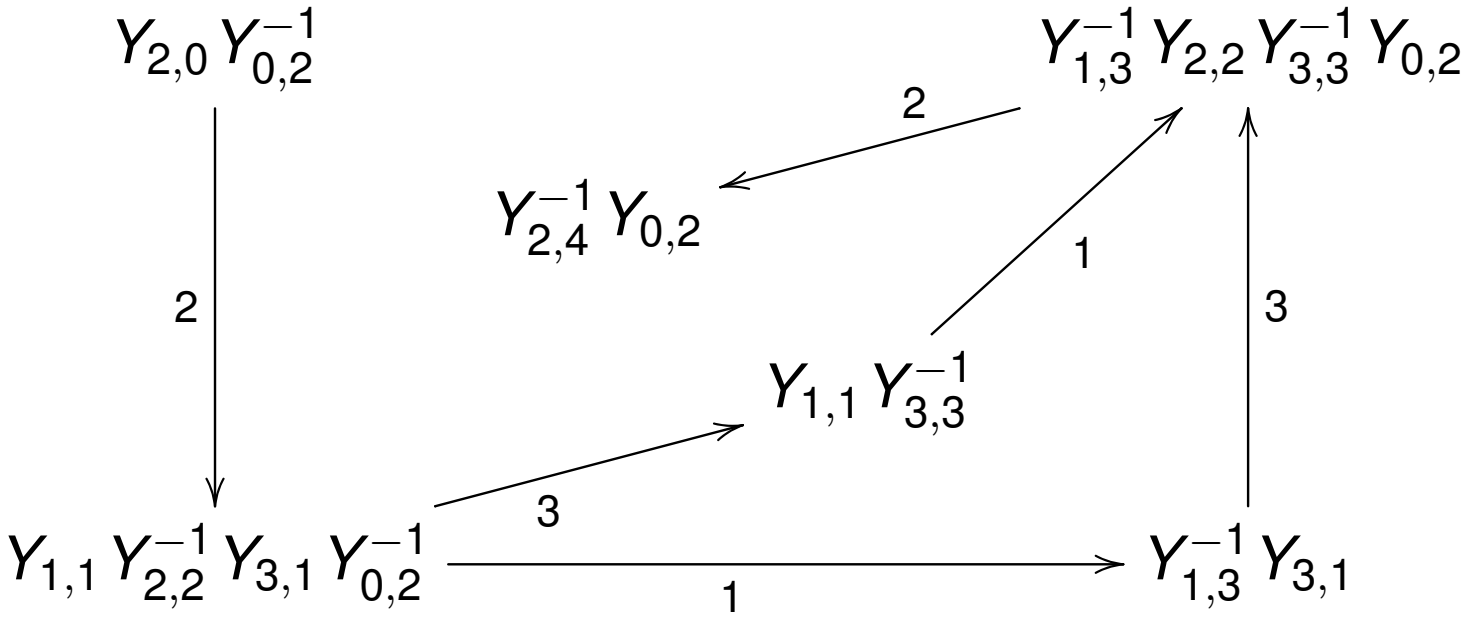
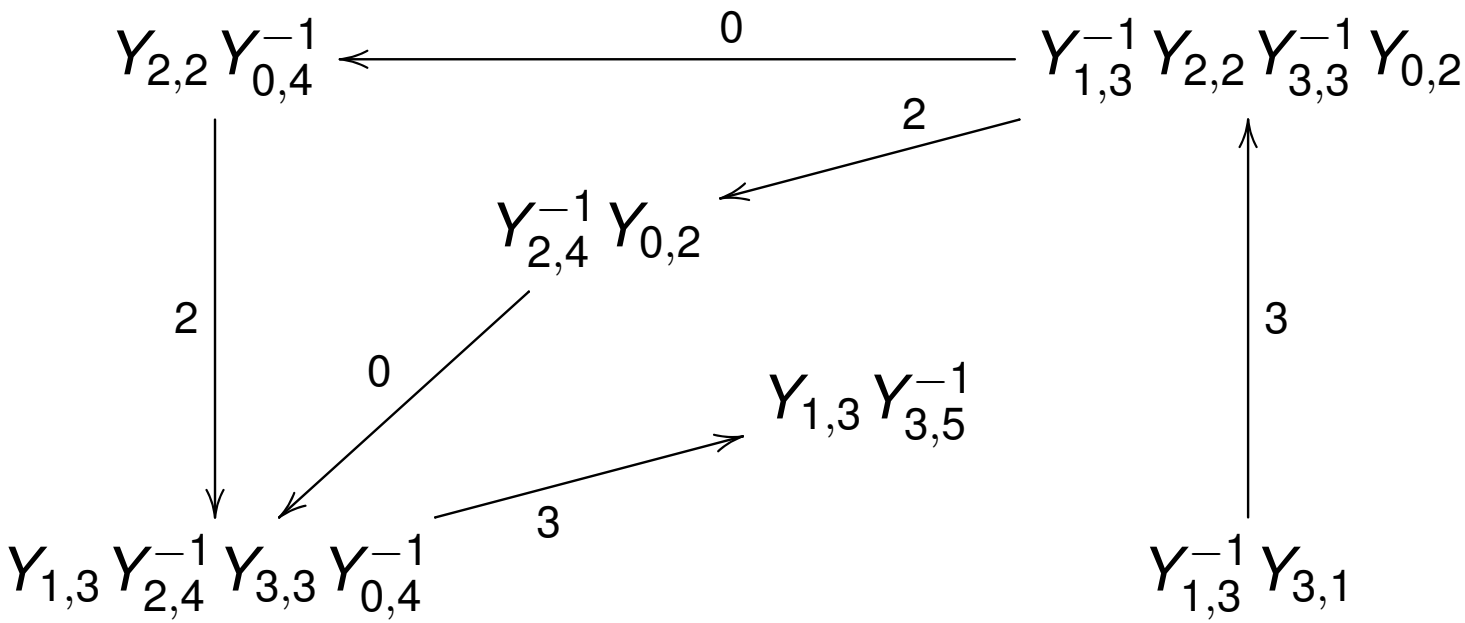
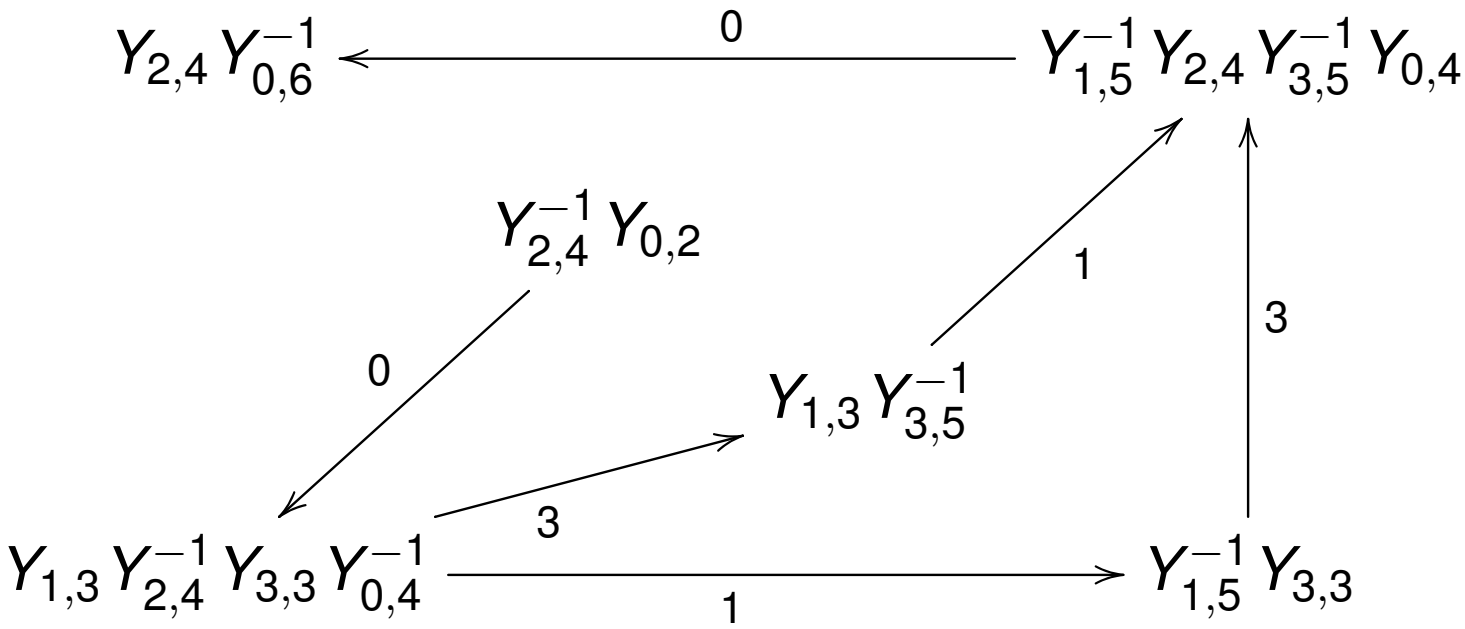
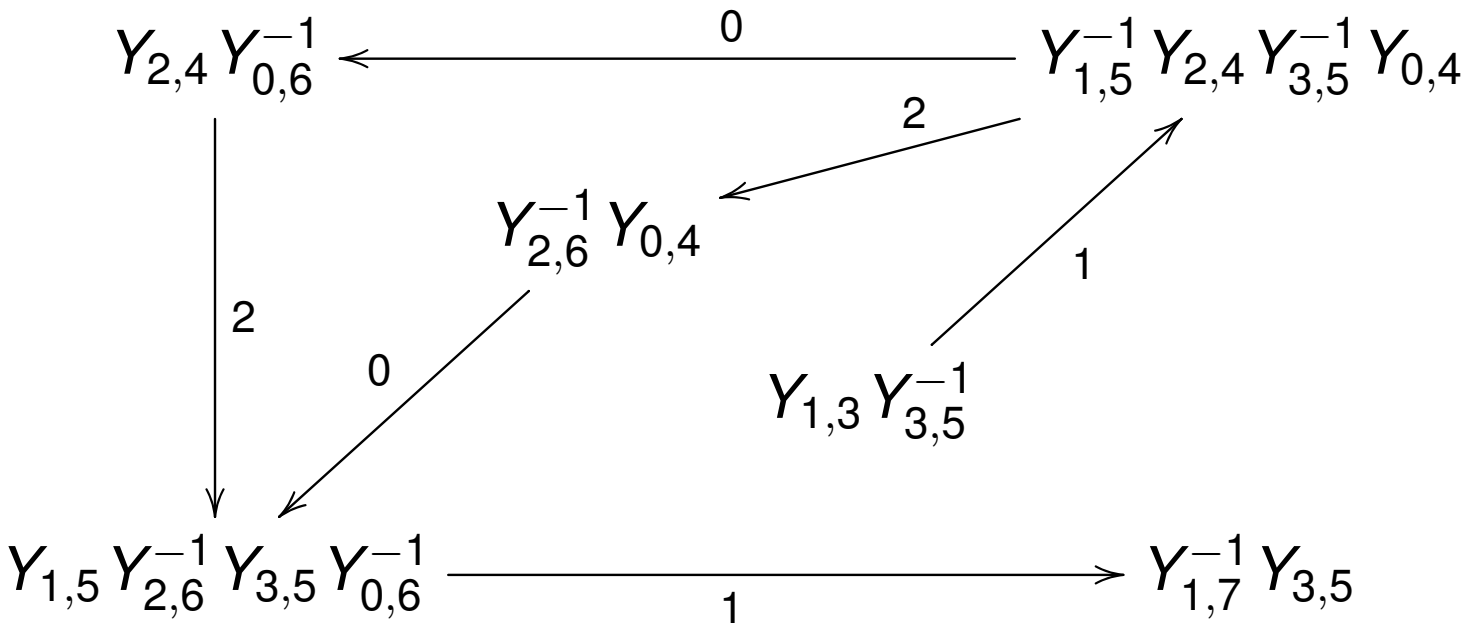
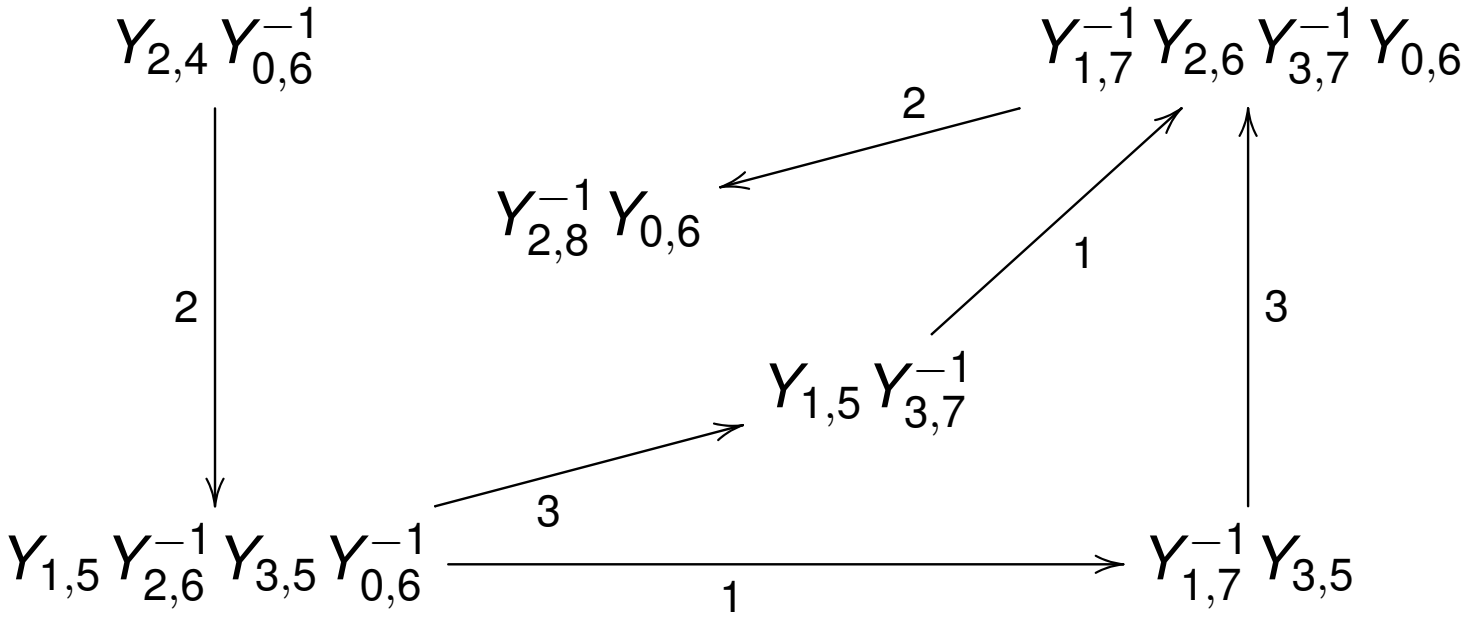

$Y_{2,0}\,Y_{0,2}^{-1}$
$Y_{1,3}^{-1}\,Y_{2,2}\,Y_{3,3}^{-1}\,Y_{0,2}$
2
$Y_{2,4}^{-1}\,Y_{0,2}$
1
2
3
$Y_{1,1}\,Y_{3,3}^{-1}$
3
$Y_{1,1}\,Y_{2,2}^{-1}\,Y_{3,1}\,Y_{0,2}^{-1}$
$Y_{1,3}^{-1}\,Y_{3,1}$
1


$Y_{2,2}\,Y_{0,4}^{-1}$
0
$Y_{1,3}^{-1}\,Y_{2,2}\,Y_{3,3}^{-1}\,Y_{0,2}$
2
$Y_{2,4}^{-1}\,Y_{0,2}$
2
0
3
$Y_{1,3}\,Y_{3,5}^{-1}$
3
$Y_{1,3}\,Y_{2,4}^{-1}\,Y_{3,3}\,Y_{0,4}^{-1}$
$Y_{1,3}^{-1}\,Y_{3,1}$


$Y_{2,4}\,Y_{0,6}^{-1}$
0
$Y_{1,5}^{-1}\,Y_{2,4}\,Y_{3,5}^{-1}\,Y_{0,4}$
$Y_{2,4}^{-1}\,Y_{0,2}$
1
3
0
$Y_{1,3}\,Y_{3,5}^{-1}$
3
$Y_{1,3}\,Y_{2,4}^{-1}\,Y_{3,3}\,Y_{0,4}^{-1}$
1
$Y_{1,5}^{-1}\,Y_{3,3}$


$Y_{2,4}\,Y_{0,6}^{-1}$
0
$Y_{1,5}^{-1}\,Y_{2,4}\,Y_{3,5}^{-1}\,Y_{0,4}$
2
$Y_{2,6}^{-1}\,Y_{0,4}$
1
2
0
$Y_{1,3}\,Y_{3,5}^{-1}$
$Y_{1,5}\,Y_{2,6}^{-1}\,Y_{3,5}\,Y_{0,6}^{-1}$
1
$Y_{1,7}^{-1}\,Y_{3,5}$


$Y_{2,4}\,Y_{0,6}^{-1}$
$Y_{1,7}^{-1}\,Y_{2,6}\,Y_{3,7}^{-1}\,Y_{0,6}$
2
$Y_{2,8}^{-1}\,Y_{0,6}$
1
2
3
$Y_{1,5}\,Y_{3,7}^{-1}$
3
$Y_{1,5}\,Y_{2,6}^{-1}\,Y_{3,5}\,Y_{0,6}^{-1}$
$Y_{1,7}^{-1}\,Y_{3,5}$
1

We define the applications

- $\tau_{2,-\delta} : \mathcal{M}_{2,a} \to \mathcal{M}_{2,a}$ by

$$\tau_{2,-\delta}\left(e^{\nu}\prod Y_{i,n}^{u_{i,n}}\right) = e^{\nu-\delta}\prod Y_{i,n+2}^{u_{i,n}},$$

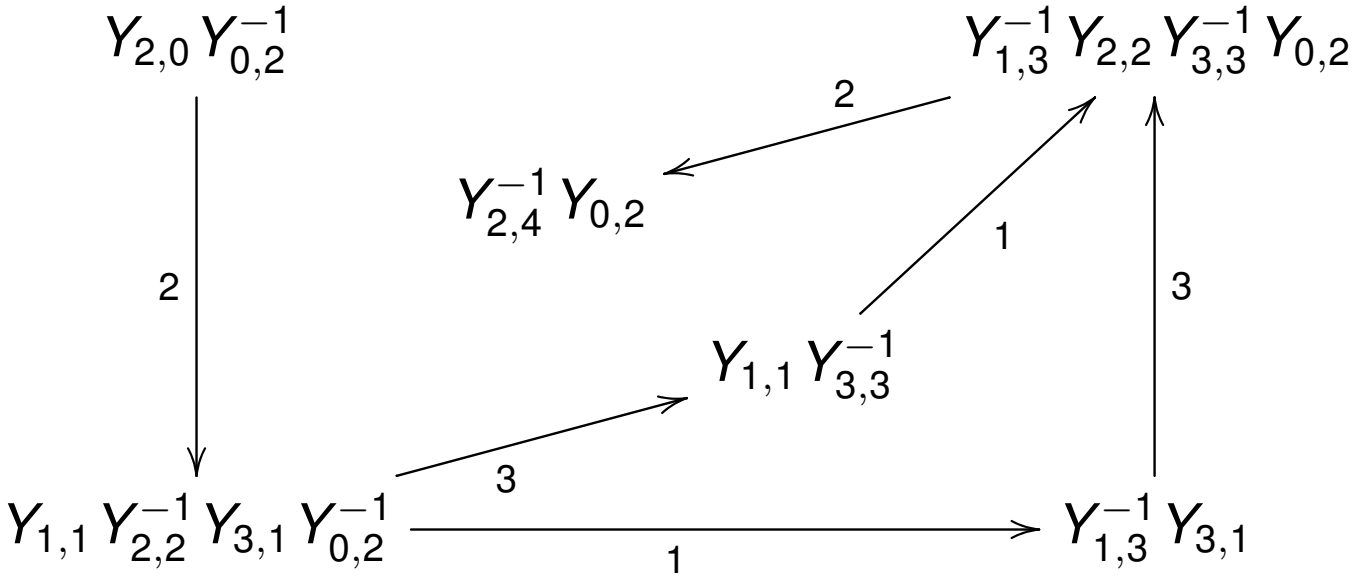
- $\phi : \mathcal{M}_{2,a} \to \mathcal{M}_{2,a}$ by

$$\phi\left(\prod Y_{i,n}^{u_{i,n}}\right) = \prod Y_{i+1,n+1}^{u_{i,n}}.$$

$\tau_{2,-\delta}$ and ϕ are automorphisms of the crystal $\mathcal{M}_{2,a}$.

Remark ϕ acts on $\mathcal{M}_{2,a}$ as a screwing. This is the promotion operator of the crystal $\mathcal{M}_{2,a}$.

Associated $\mathcal{U}_q^{\nu,0}(sl_4^{tor})$-module

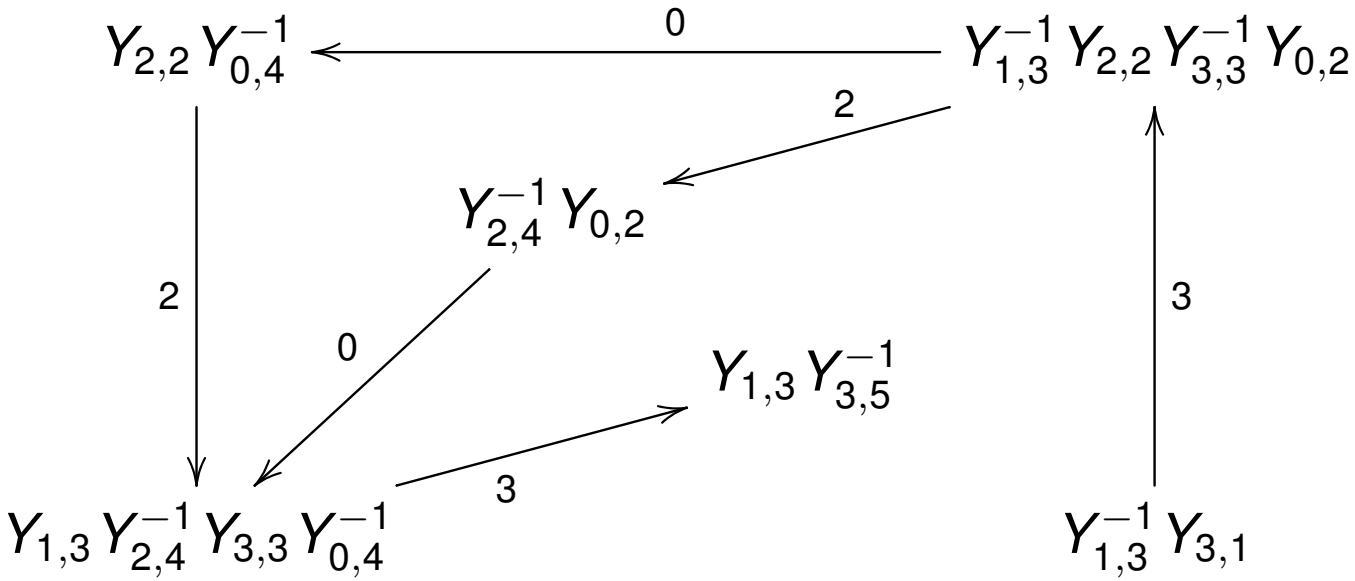


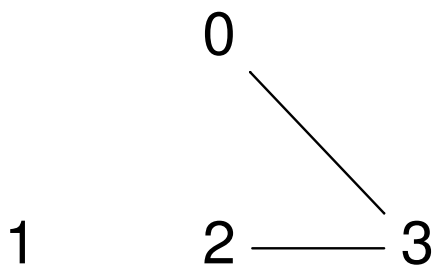
0

$$\chi_q(V_0(Y_{2,0})) = Y_{2,0} + Y_{1,1}\,Y_{2,2}^{-1}\,Y_{3,1} + Y_{1,1}\,Y_{3,3}^{-1} + Y_{1,3}^{-1}\,Y_{3,1} + Y_{1,3}^{-1}\,Y_{2,2}\,Y_{3,3}^{-1} + Y_{2,4}^{-1}$$

1 —— 2 —— 3

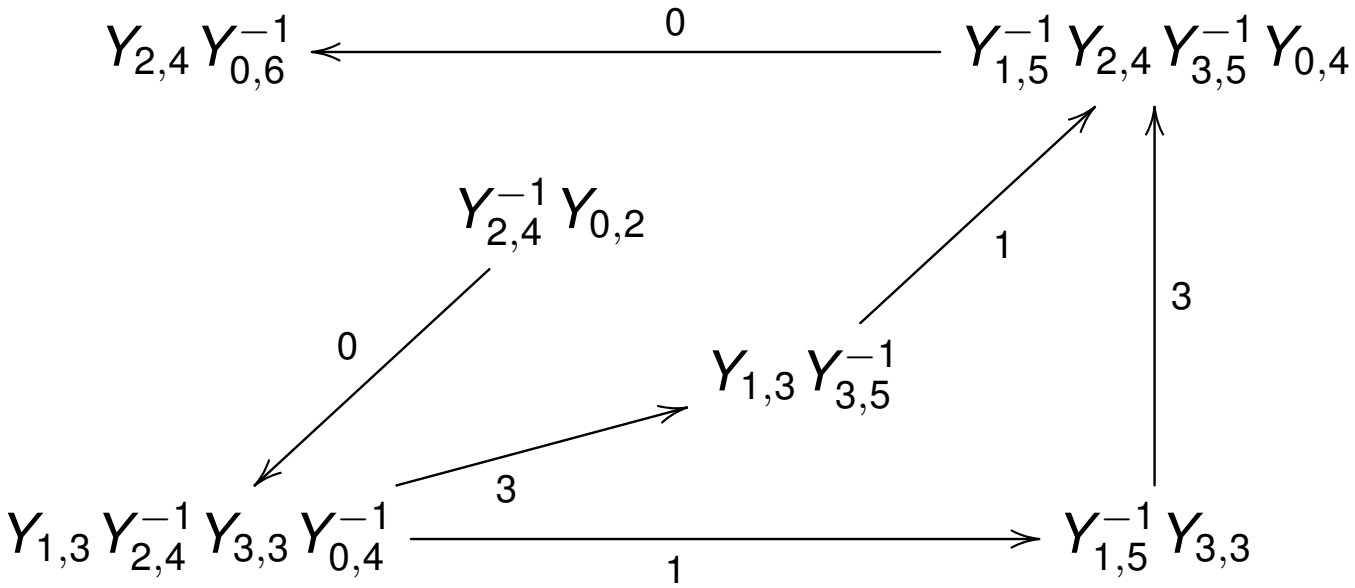
Associated $\mathcal{U}_q^{\nu,1}(sl_4^{tor})$-module

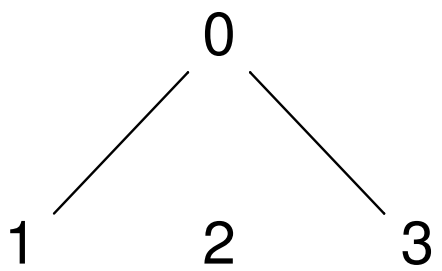




$$\chi_q(V_1(Y_{3,1})) = Y_{3,1} + Y_{2,2}\,Y_{3,3}^{-1}\,Y_{0,2} + Y_{2,2}\,Y_{0,4}^{-1} + Y_{2,4}^{-1}\,Y_{0,2} + Y_{2,4}^{-1}\,Y_{3,3}\,Y_{0,4}^{-1} + Y_{3,5}^{-1}$$

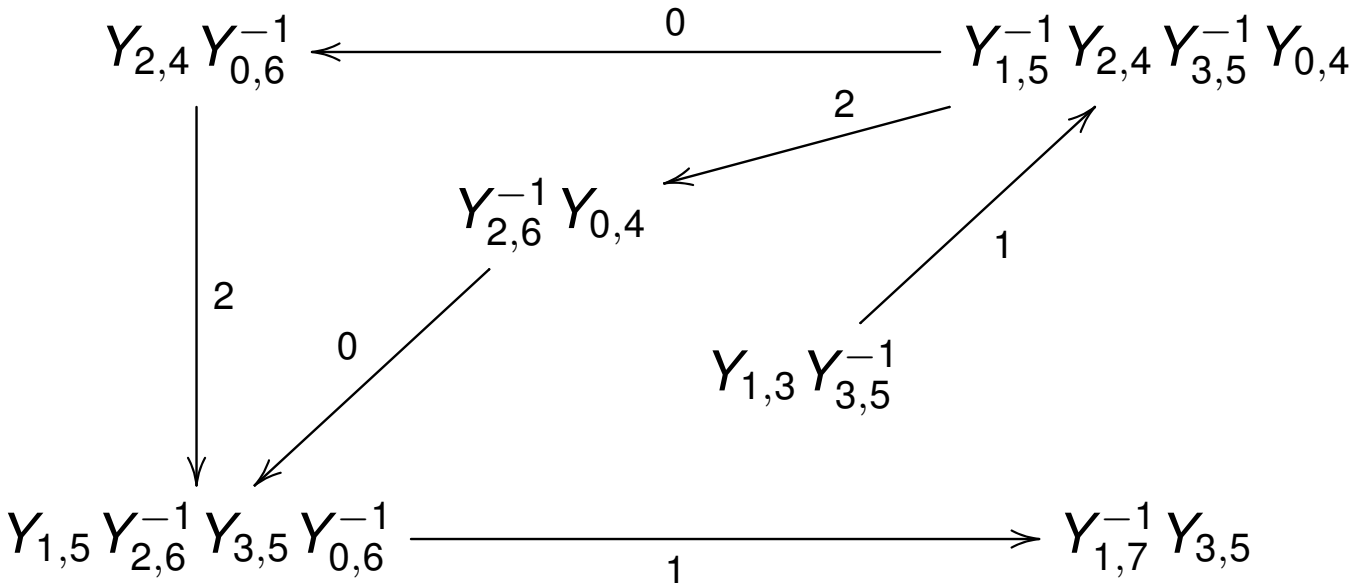
Associated $\mathcal{U}_q^{\nu,2}(sl_4^{tor})$-module

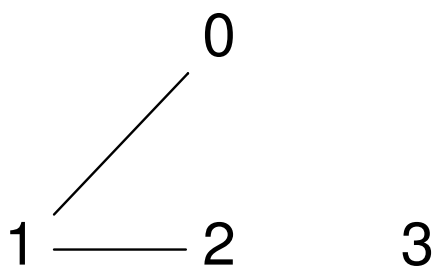




$$\chi_q(V_2(Y_{0,2})) = Y_{0,2} + Y_{1,3}\,Y_{3,3}\,Y_{0,4}^{-1} + Y_{1,5}^{-1}\,Y_{3,3} + Y_{1,3}\,Y_{3,5}^{-1} + Y_{1,5}^{-1}\,Y_{3,5}^{-1}\,Y_{0,4} + Y_{0,6}^{-1}$$

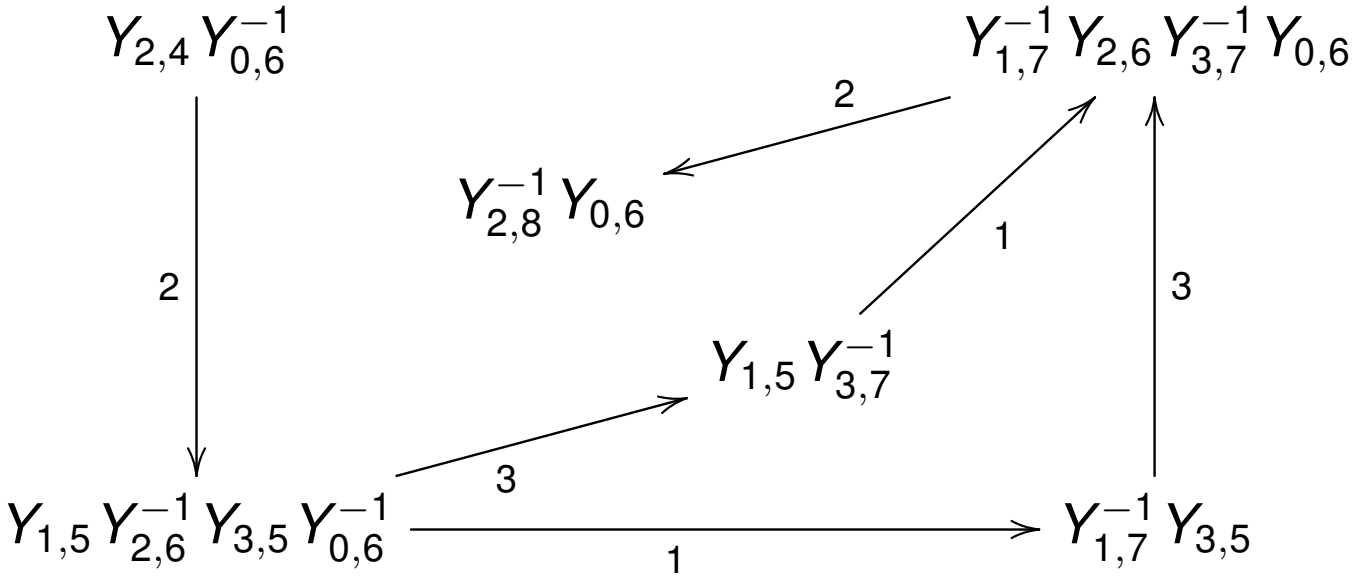
Associated $\mathcal{U}_q^{\nu,3}(sl_4^{tor})$-module





$$\chi_q(V_3(Y_{1,3})) = Y_{1,3} + Y_{1,5}^{-1}\,Y_{2,4}\,Y_{0,4} + Y_{2,6}^{-1}\,Y_{0,4} + Y_{2,4}\,Y_{0,6}^{-1} + Y_{1,5}\,Y_{2,6}^{-1}\,Y_{0,6}^{-1} + Y_{1,7}^{-1}$$

Associated $\mathcal{U}_q^{\nu,0}(sl_4^{tor})$-module



0

1 —— 2 —— 3

$$\chi_q(V_0(Y_{2,4})) = Y_{2,4} + Y_{1,5}\,Y_{2,6}^{-1}\,Y_{3,5} + Y_{1,5}\,Y_{3,7}^{-1} + Y_{1,7}^{-1}\,Y_{3,5} + Y_{1,7}^{-1}\,Y_{2,6}\,Y_{3,7}^{-1} + Y_{2,8}^{-1}$$