

Monomial crystals and promotion operators

The monomial crystal $\mathcal{M}(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1})$

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We describe the monomial crystal $\mathcal{M} = \mathcal{M}(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1})$ of type A_3 used in arXiv :1207.3299. Let $\phi : \mathcal{M} \rightarrow \mathcal{M}$ be defined by

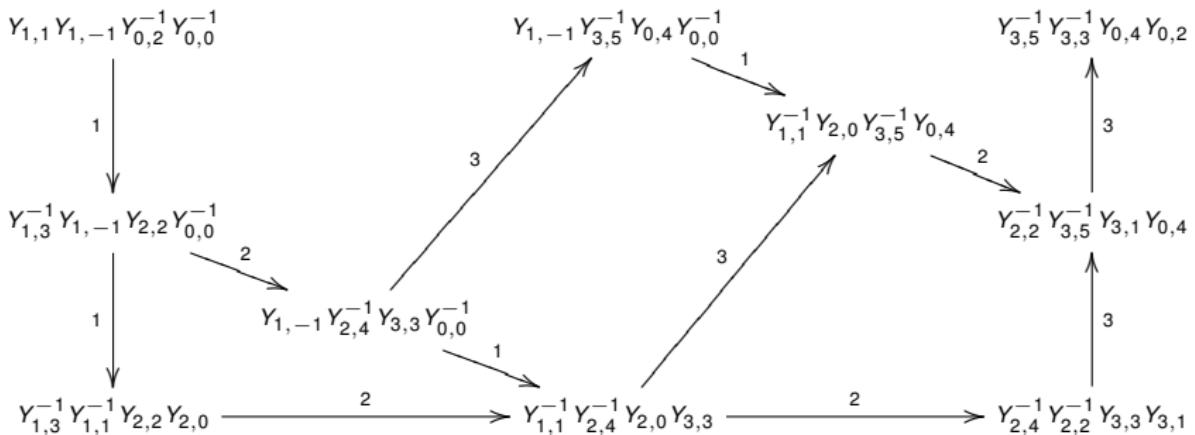
$$\phi\left(\prod Y_{i,n}^{u_{i,n}}\right) = \prod Y_{i+1,n+1}^{u_{i,n}}.$$

We give

- the sub- I_j -crystals $\mathcal{M}_j = \mathcal{M}_{I_j}(\phi^j(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1}))$ for $j = 0, \dots, 4$,
- the corresponding Dynkin diagram,
- the $\mathcal{U}_q^{\nu,j}(sl_4^{tor})$ -module associated to \mathcal{M}_j whose q -character is

$$\Xi^j\left(\sum_{m \in \mathcal{M}_j} m\right).$$

The action of the promotion operator ϕ on \mathcal{M} can be viewed as a screwing.



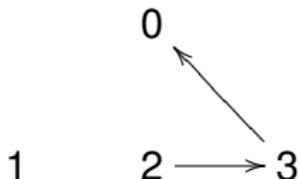
0

Associated $\mathcal{U}_q^{\nu,0}(sl_4^{tor})$ -module :

1 → 2 → 3

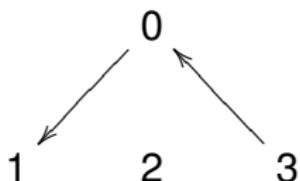
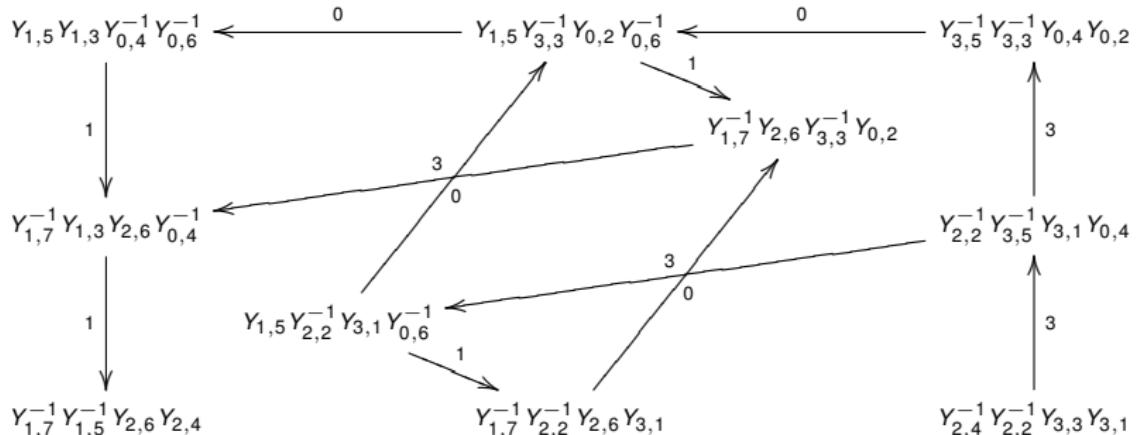
 $V(\Xi^0(Y_{1,1}Y_{1,-1}Y_{0,2}^{-1}Y_{0,0}^{-1}))$

$$\begin{array}{ccccc}
Y_{1,5} Y_{1,3} Y_{0,4}^{-1} Y_{0,6}^{-1} & \xleftarrow{0} & Y_{1,5} Y_{3,3}^{-1} Y_{0,2} Y_{0,6}^{-1} & \xleftarrow{0} & Y_{3,5}^{-1} Y_{3,3}^{-1} Y_{0,4} Y_{0,2} \\
& \nearrow & \downarrow & \nearrow & \uparrow 3 \\
Y_{1,1}^{-1} Y_{1,5} Y_{2,0} Y_{0,6}^{-1} & \xleftarrow{2} & 0 & \xrightarrow{3} & Y_{2,2}^{-1} Y_{3,5}^{-1} Y_{3,1} Y_{0,4} \\
& \searrow & \downarrow & \nearrow & \uparrow 3 \\
Y_{1,5} Y_{2,2}^{-1} Y_{3,1} Y_{0,6}^{-1} & \xleftarrow{2} & 0 & \xrightarrow{3} & Y_{1,1}^{-1} Y_{2,4}^{-1} Y_{2,0} Y_{3,3} \\
& \nearrow & \downarrow & \nearrow & \uparrow 3 \\
Y_{1,3}^{-1} Y_{1,1}^{-1} Y_{2,2} Y_{2,0} & \xrightarrow{2} & Y_{1,1}^{-1} Y_{2,4}^{-1} Y_{2,0} Y_{3,3} & \xrightarrow{2} & Y_{2,4}^{-1} Y_{2,2}^{-1} Y_{3,3} Y_{3,1}
\end{array}$$



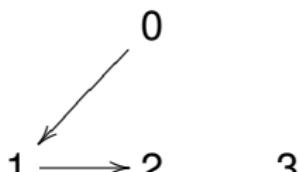
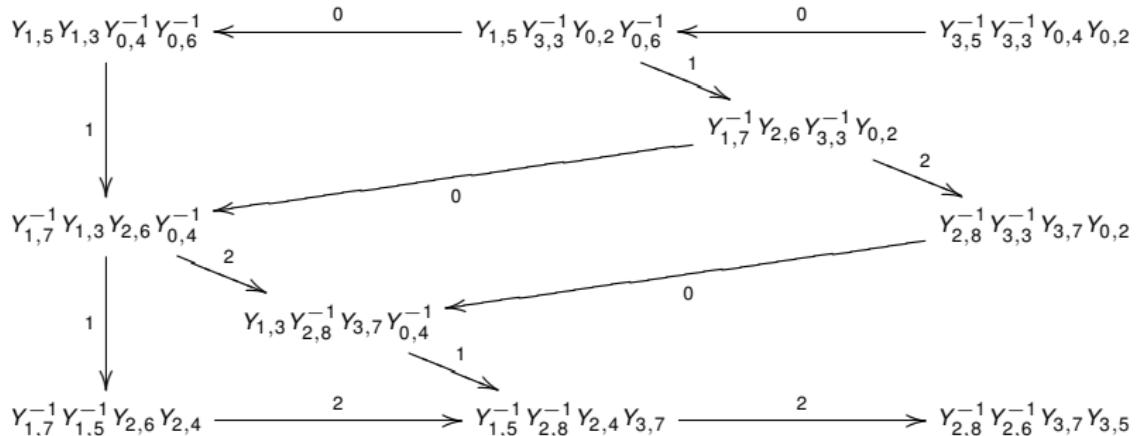
Associated $\mathcal{U}_q^{V,1}(sl_4^{tor})$ -module :

$$V(\Xi^1(Y_{1,3}^{-1} Y_{1,1}^{-1} Y_{2,2} Y_{2,0}))$$



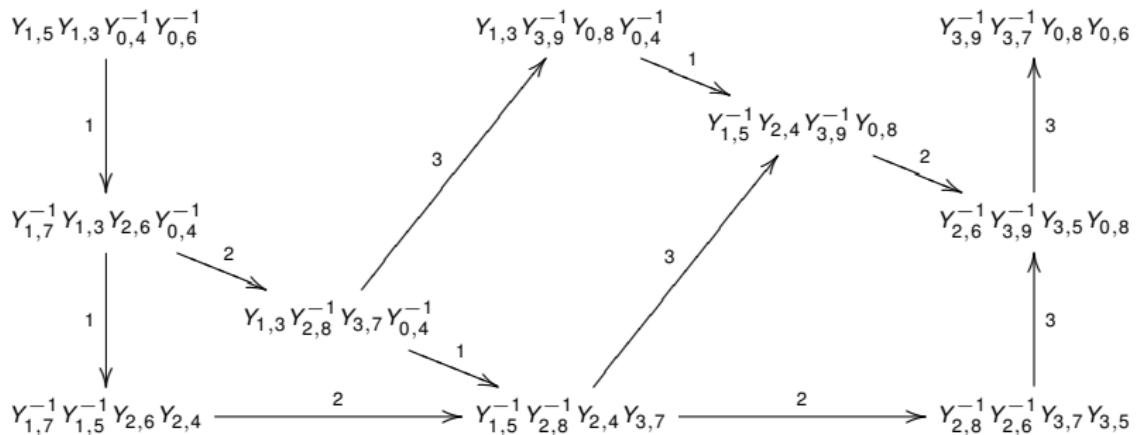
Associated $\mathcal{U}_q^{v,2}(sl_4^{tor})$ -module :

$$V(\Xi^2(Y_{2,4}^{-1}Y_{2,2}^{-1}Y_{3,3}Y_{3,1}))$$



Associated $\mathcal{U}_q^{\nu,3}(sl_4^{tor})$ -module :

$$V(\Xi^3(Y_{3,5}^{-1}Y_{3,3}^{-1}Y_{0,4}Y_{0,2}))$$



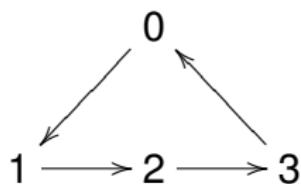
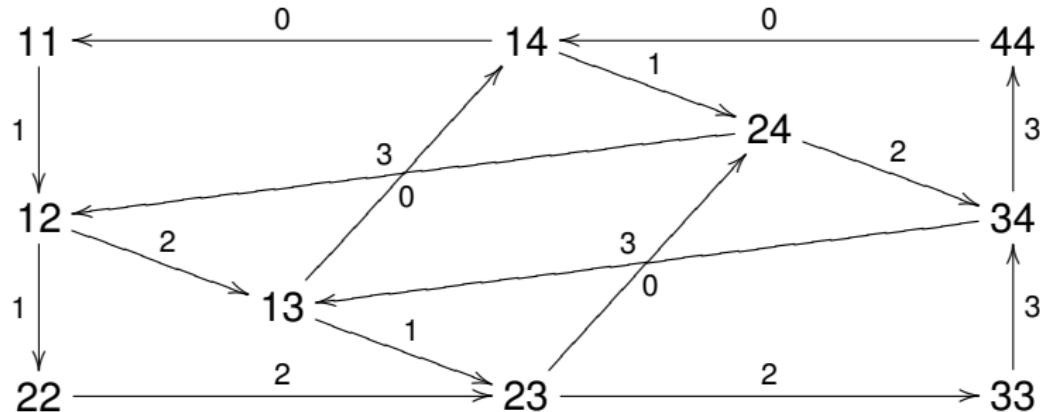
Associated $\mathcal{U}_q^{\nu,0}(sl_4^{tor})$ -module :

$$1 \longrightarrow 2 \longrightarrow 3$$

$$V(\Xi^0(Y_{1,5} Y_{1,3} Y_{0,4}^{-1} Y_{0,6}^{-1}))$$

Link with the P_{cl} -crystal $\mathcal{B}(2\Lambda_1)$

Let us give the P_{cl} -crystal $\mathcal{B}(2\Lambda_1)$ of type $A_3^{(1)}$ obtained by promotion operator pr . In the crystal, $i \downarrow j = \boxed{i \mid j}$ for all i, j .



Crystal graph of the
 $\mathcal{U}_q(\hat{\mathfrak{sl}}_4)$ -module $W(2\varpi_1)$